Progression towards a written method for division

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction
- sharing into equal amounts

They also need to understand and work with certain principles, i.e. that it is:

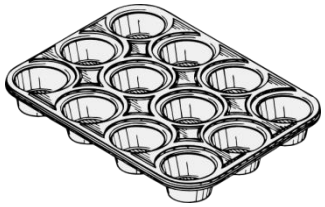
- the inverse of multiplication
- not commutative i.e. 15 ÷3 is not the same as 3 ÷ 15
- not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2

RECEPTION

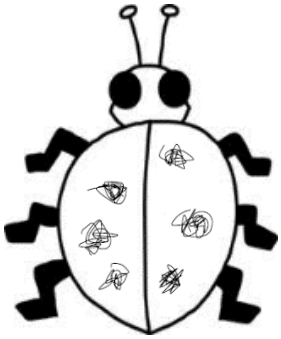
Early Learning Goal:
Children solve problems, including halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.

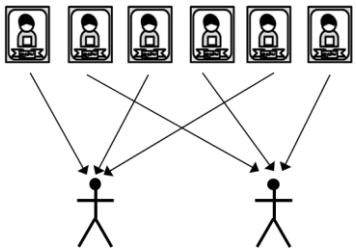
A child's jotting showing how they shared the apples at snack time between two groups.

Y1

End of Year Objective:
Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two

people, how many do they each get?’ They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



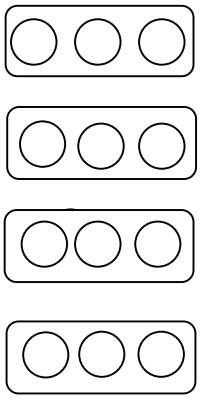
Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as ‘… left over’.

Y2

End of Year Objective:
Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.

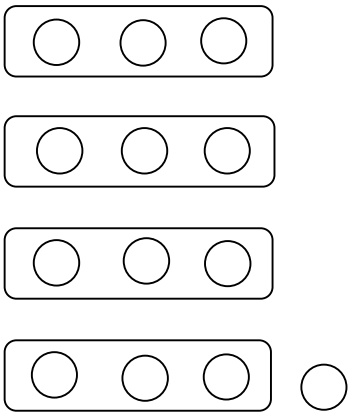
Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation with groups **underneath each other**, e.g.

12 ÷ 3 =



Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'
They should also continue to develop their knowledge of division with remainders, e.g

13 ÷ 4 =



13 ÷ 4 = 3 remainder 1

Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation 13 ÷ 4, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?
Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?
Answer: 4 (the remaining 1 apple still needs to be placed into a box)

Y3

End of Year Objective:
Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

43 ÷ 8 being interpreted as how many groups of 8 are there in 43**? (organised one underneath the other as shown in Year 2)**

I know that there are five 8s in 40 so, 43 ÷ 8 = 5 remainder 3

The children may begin to divide numbers where the dividend is beyond their tables knowledge.
E.g. 56 ÷ 4 = 14
Some more able children may be encouraged to make use of known mental facts for this. In this case thinking of 56 as being made up of 40 and 16 and knowing that 40 ÷ 4 = 10 and 16 ÷ 4 = 4

The teacher will use their professional judgement to make a decision about when children may see the traditional layout of division:

$$\begin{array}{r} 14 \\ 4\overline{)56} \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

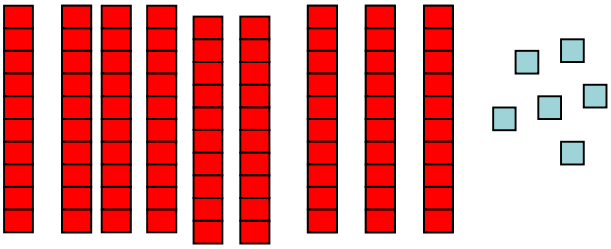
Y4

End of Year Objective:
Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

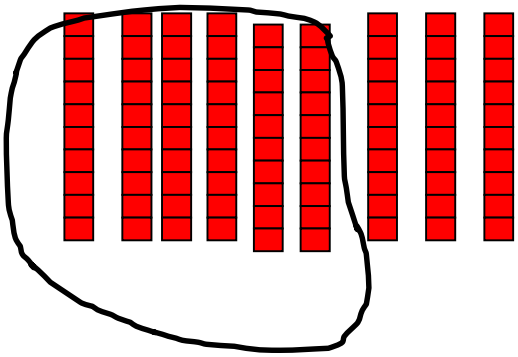
Progression in Y4 will begin with ensuring the children understand how the formal method works.

To calculate 96 ÷ 6:

Represent 96 using base 10 materials.

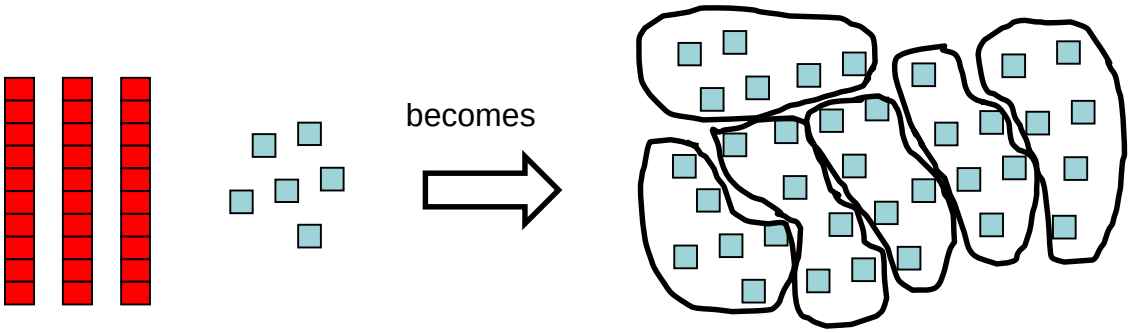


How many groups of 6 can be made out of the tens (without changing their form)? Answer **1** group



$$6\overline{)96}\quad 1$$

There are 3 tens left over so they need to be exchanged for units (ones) leaving:



How many groups of 6 can be made out of the 36 units (ones)? Answer **6** groups.

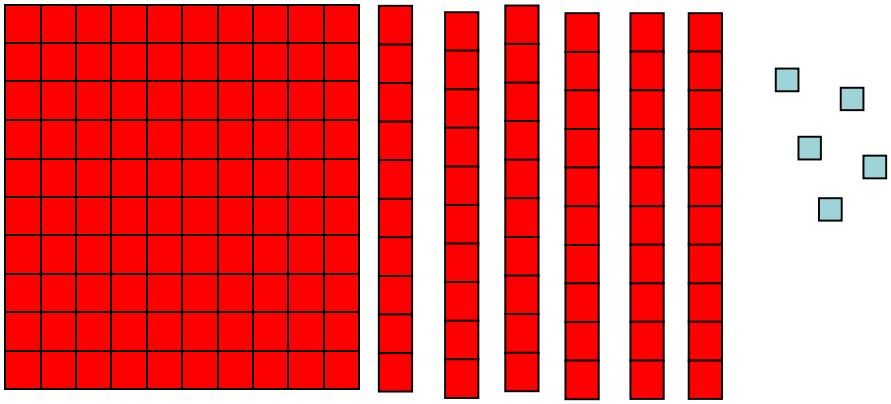
$$6\overline{)96}\quad 16$$

This will progress onto dividing a 3 digit number by a 1 digit number.

E.g.

165 ÷ 5

Would be modelled using base 10 materials.

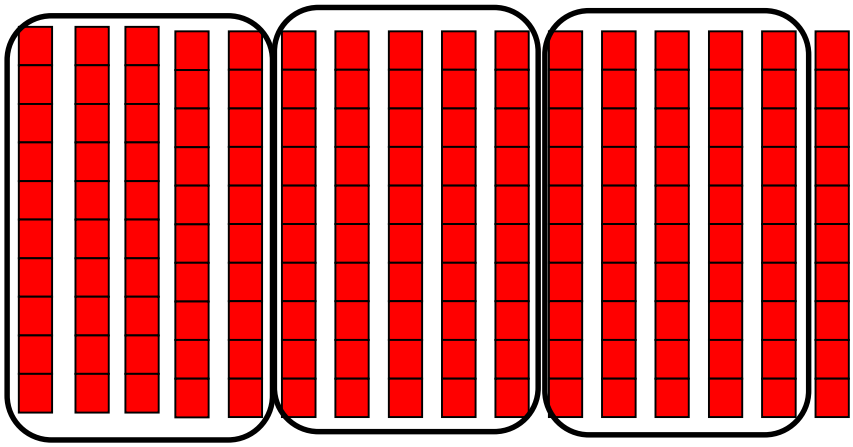


Again this would be explained as how many groups of 5 can I make out of the hundreds without changing their form? Answer none.
So they need to be exchanged for 10 tens.

$$5\overline{)165}\quad 0$$

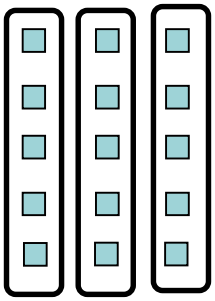
This will leave us with 16 tens.

How many groups of 5 can be made out of the 16 tens without changing their form? Answer 3 with 1 ten left over.



$$5\overline{)16^{1}5}\quad 03$$

This ten will be exchanged for units (ones) and added to the 5 we started with giving us 15 units.



$$5\overline{)16^{1}5}\quad 033$$

How many groups of 5 can we make out of the 15 units without changing their form? Answer 3

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Y5

End of Year Objective:
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Base 10 materials may continue to be used to model the method of division where necessary. Children will develop their skills with increasingly big numbers including calculations where there is exchange from the first number and then where there is exchange within the number. I.e. no carry from thousands to hundreds in the example underneath.

E.g.

$$\begin{array}{r} 72 \\ 6\overline{)4\,3^{1}2} \end{array} \qquad \begin{array}{r} 1271 \\ 4\overline{)5\,{}^{1}0\,{}^{2}8\,4} \end{array} \qquad \begin{array}{r} 1054 \\ 7\overline{)7\,3^{3}7^{2}8} \end{array}$$

Y6

End of Year Objective:
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Use written division methods in cases where the answer has up to two decimal places.

To develop the formal method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

6367 ÷ 28

$$\begin{array}{r} 227\text{r}11 \\ 28\overline{)6367} \\ 56\downarrow \\ \hline 76 \\ 56\downarrow \\ \hline 207 \end{array}$$

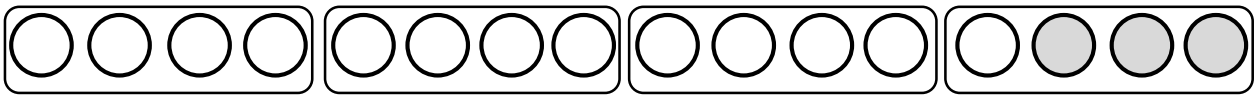
The starting point would to be to write out the factors of the divisor, in this case 28.
This will be interpreted as: how many groups of 28 can be made out of 63 (63 hundreds)? Answer 2 (28 X 2 = 56) This will leave 7 (hundreds) to be carried into the tens and exchanged for 70 tens.
This is added to the 6 tens we already have.
How many groups of 28 can be made out of the 76 tens? Answer 2 (28 X 2 = 56) This will leave 20 (tens) to be carried into the units.
This is added to the 7 units we already have giving 207 units in total to be divided by 28.
How many groups of 28 can be made out of 207 units? Answer 7 remainder 11(28 X 7 = 196)

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the division method and solve calculations interpreting the remainder as a decimal up to two decimal places.

This should first be demonstrated using a simple calculation such as 13 ÷ 4 to show the remainder initially as a fraction.
It can be explained like this:
I can make 3 groups of 4 with 13, leaving a remainder of 1. (whole number remainder)
This remainder is part of a group of 4 and therefore as a fraction is ¼. (fraction remainder)
As a decimal this is a remainder of 0.25. (decimal remainder)

So: 13 ÷ 4 = 3 r1
13 ÷ 4 = 3 r ¼
13 ÷ 4 = 3.25
Modelled thus: (this may be modelled in the same way as Year 2)



3574 ÷ 8

$$\begin{array}{r} 4\ 4\ 6\ r\ 6 \\ 8\overline{)3\ 5\ {}^{3}7\ {}^{5}4} \end{array}$$

$\frac{6}{8}$ ← remainder / divisor

So 3574 ÷ 8 is $446\frac{6}{8}$
(when the remainder is shown as a fraction)

$$\begin{array}{r} 4\ 4\ 6\ .\ 7\ 5 \\ 8\overline{)3\ 5\ {}^{3}7\ {}^{5}4\ .\ {}^{6}0\ {}^{4}0} \end{array}$$

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2}$ = 0.5

Quarters: $\frac{1}{4}$ = 0.25, $\frac{3}{4}$ = 0.75

Fifths: $\frac{1}{5}$ = 0.2, $\frac{2}{5}$ = 0.4, $\frac{3}{5}$ = 0.6, $\frac{4}{5}$ = 0.8

Tenths: $\frac{1}{10}$ = 0.1, $\frac{2}{10}$ = 0.2, $\frac{3}{10}$ = 0.3, $\frac{4}{10}$ = 0.4, $\frac{5}{10}$ = 0.5, $\frac{6}{10}$ = 0.6, $\frac{7}{10}$ = 0.7, $\frac{8}{10}$ = 0.8, $\frac{9}{10}$ = 0.9

and reduce other equivalent fractions to their lowest terms.

In the example above, 3574 ÷ 8, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75